Sample windows in Asynchronous Efficiency measurements

Abstract

A drive for the increased efficiency of electronic power conversion systems requires greater attention to the impact of sampling techniques that are instinctively utilised, yet commonly inappropriate for dynamic testing environments.

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1 INTRODUCTION

Analysis of modern power systems will often require an efficiency measurement between two asynchronous, periodic waveforms. A typical example would be a variable-frequency drive that will take a mains input and generate a secondary AC output with a selectable frequency, so that the speed of an AC motor can be controlled.

In order to quickly and accurately establish system efficiency, the correct sampling window for both input and output measurements should be established by identifying the frequency of input and output waveforms. Despite this fact, the approach of synchronising to each waveform independently is not typical in many power analyzers or data acquisition systems.

This document seeks to explain why independent synchronisation is not typical and to illustrate why all users whose objective is to minimise error in efficiency measurements, should seek independent frequency detection and associated windows when measuring asynchronous waveforms.

1.1 DOCUMENT STRUCTURE

Section 2 introduces the limitations in using a finite window to measure a Root Mean Square (RMS) value and the main strategies for minimising the error for a single waveform.

Section 3 extends the concepts of windowing to an efficiency measurement in an asynchronous system.

2 SAMPLE WINDOW SIZE – MEASUREMENT ERROR

Periodic waveforms are usually quantified in time-average measurement. The most commonly used unit of measurement for AC power electronics is RMS, the Root Mean Square value.

2.1 RMS definition of a periodic function

The RMS of a time dependant signal is defined as[1]:

$$V = \sqrt{\frac{1}{T}\int_{T_0}^{T_0+T} v^2(t)\,dt}$$

Where:

$$T = Window\ wize$$

$$T_0 = Window\ start\ time$$

For any periodic signal, the determination of true RMS requires a measurement window size equal to an integer number of periods.

2.2 QUANTIFYING WINDOW ERROR

A common approach in data acquisition systems or Digital Multi Meters (DMM) is to use a fixed sized sample window.

Fixing the window makes development of the measurement device easier as there is a constrained buffer size requirement and known response time for a measurement to be returned. However, this approach introduces a windowing error.

2.2.1 Window error

Taking the simplest AC signal as an example, a single sine wave, we can estimate the error from an imperfect measurement window.

Given a sine wave with a magnitude $\sqrt{2}$ (which means that the true RMS is 1):

$$v(t) = \sqrt{2}\sin(t)$$

The RMS is therefore defined as:

$$RMS_{measured} = \sqrt{\frac{1}{T}\int_{T_0}^{T_0+T} (\sqrt{2}\sin(t))^2\ dt}$$

Note that T_0 can be varied to simulate the window shifting from the zero-crossing point, since fixed time windows cannot be guaranteed to be synchronous with a specific point in a cycle.

[1] This is the continuous definition. Real measurements will involve discrete samples but for systems with high sample rates, the error introduced by discrete measurement will be relatively small and have been excluded from this analysis

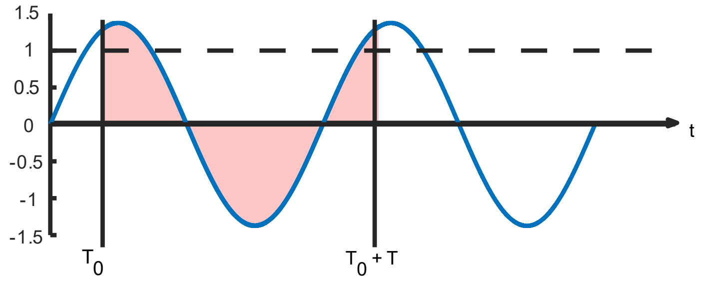

Figure 1: Time limited integration function

Integration over a defined time period yields the following equation:

$$RMS_{measured} = \sqrt{\frac{1}{T}\left(T - \frac{\sin(2(T_0 + T))}{2} + \frac{\sin(2T_0)}{2}\right)}$$

2.2.2 Visualising the error

The following plot displays the change in measured RMS value resulting from a varying data window size and its phase alignment with the waveform. Along the y-axis, the window size T is varied and each line represents a different phase displacement from 0 to 180 degrees.

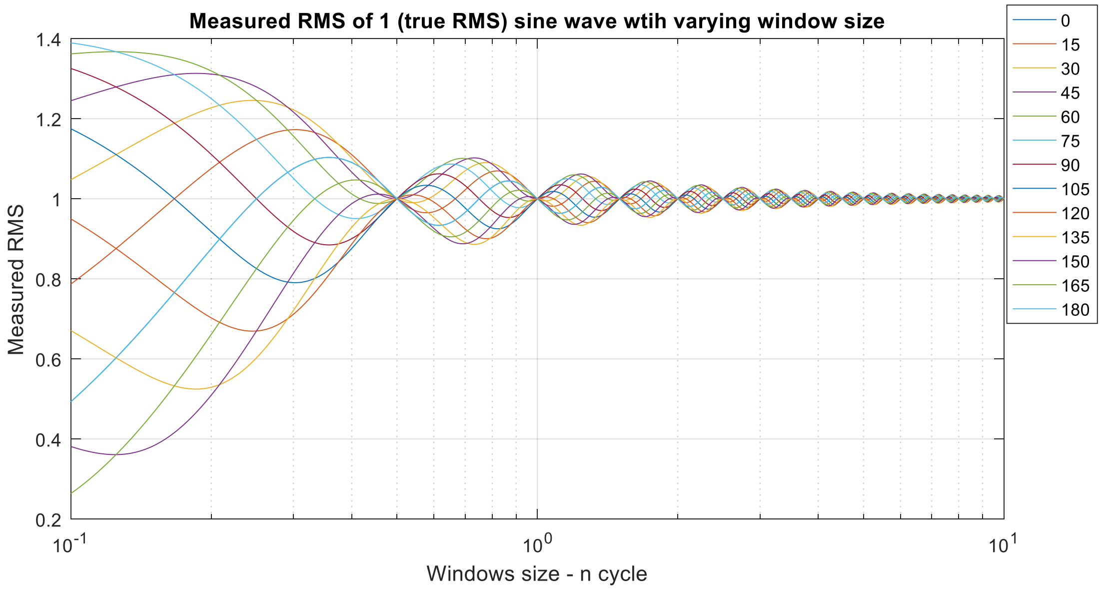

Figure 2:RMS measurement vs window size and waveform phase offset

This figure shows that for any given window size there is an upper and lower bound on the measurement error which is also influenced by window position relative to the cycle.

2.2.3 Quantifying the error limits

Absolute limits can be quantified for any window size by taking the partial derivate of the measurement error with respect to the phase offset T_0 and then identifying the functions turning points.

Which gives the solutions:

$$T = n\pi \quad and \quad n \neq 0$$

$$T_0 = \frac{m\pi - T}{2}$$

From this, we can derive the area of potential measurement error:

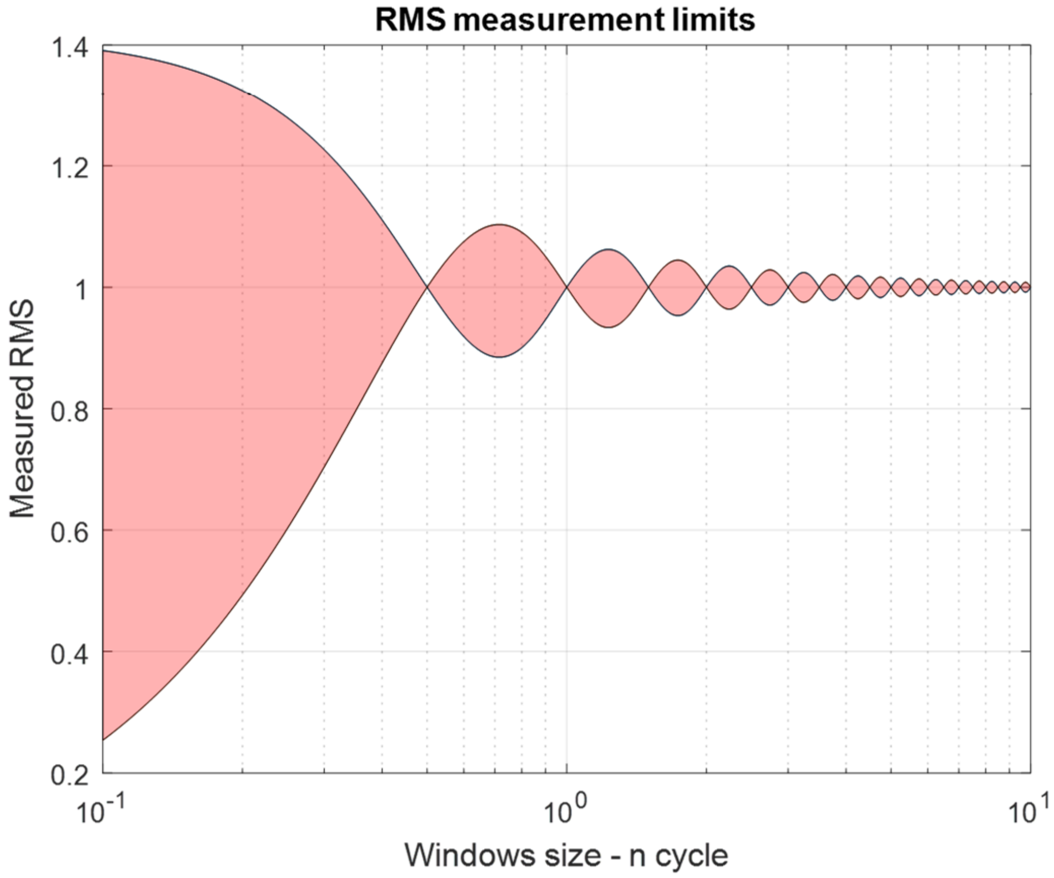

Figure 3:Measured RMS error limits for a given window size

The shape of this plot intuitively makes sense because:

- When the window is a multiple of half cycles, the solution converges to the correct answer – this is a half cycle because the waveform is squared, which therefore results in a waveform of twice the frequency.
- As the window size tends towards zero the error is bound by two scenarios: Effectively converging on either the peak or the zero crossing. Therefore, for this wave, the limits to the measurement are $0 \leq RMS_{measured} \leq \sqrt{2}$

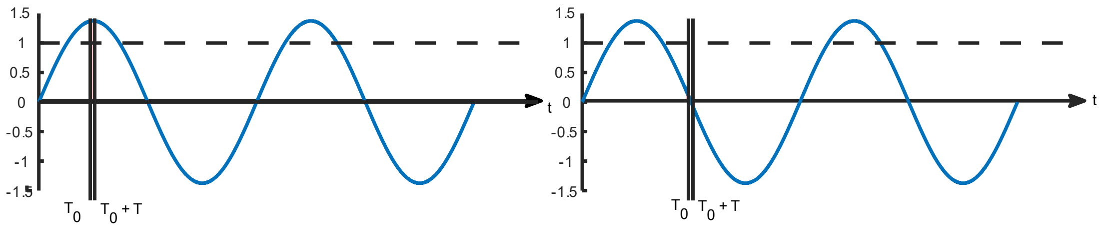

Figure 4: Upper and lower error limits for under-sized window

- It is also clear that as the size of the window increases the measurement will tend towards the correct answer. This is because the integral will include an increasing number of full cycles, plus an error term which will represent a decreasing proportion of the measurement.

2.3 MINIMISING WINDOW ERROR

Figure 3 illustrates limits on the error given for a range of window sizes.

Given an aim of measuring the correct RMS, there are two approaches to maximising the accuracy of a practical measurement:

1. Increase the window size – therefore averaging out the error inherent in non-ideal measurement.
2. Optimise the window so that it is synchronised with the periodic waveform being measured.

2.3.1 Time averaging

Increasing the windows, or averaging of multiple smaller windows, will reduce this error. This is the most common approach taken for example, by Digital Multi Meters (DMM).

This method only works if two criteria are met: first, the signal must be stable, otherwise you are no longer measuring a true cycle but are averaging the response, and; second, given that a fixed window size is selected, there is the requirement to average over multiple cycles. This introduces a frequency floor below which the accuracy cannot be guaranteed. Typically, in DMMs this will be about 20Hz to cover common line power applications.

This approach gives the benefit of measurements at fixed intervals but faces the inherent problem that it is dependant up time average and will therefore smooth out power events and needs a settling time.

It follows that while this technique may be suitable for precision measurements in steady state systems, it will be unreliable in dynamic or low frequency scenarios.

2.3.2 Window synchronisation

Measurement errors can be minimised by ensuring that the window length is always an integer multiple of cycles.

Given a correctly sized window the measured RMS can be used without additional time averaging. This has two key benefits:

- Short events, occurring on a small number of cycles, can be observed and won't be smoothed out to the same extent.
- Given the ability to track a dynamic frequency, the windows size can be adjusted live to give the answer required.

A secondary benefit to always obtaining the frequency is that additional functions can be performed, such as a Discrete Fourier Transform (DFT), which can be used to analyse the harmonics of the waveform. This is often useful for the analysis of power systems.

The following diagram illustrates RMS measurements derived from a simple 50Hz sinewave signal; first with a correctly sized window of 20ms (50Hz) and second, with a non-synchronous window of 18.18ms (55Hz).

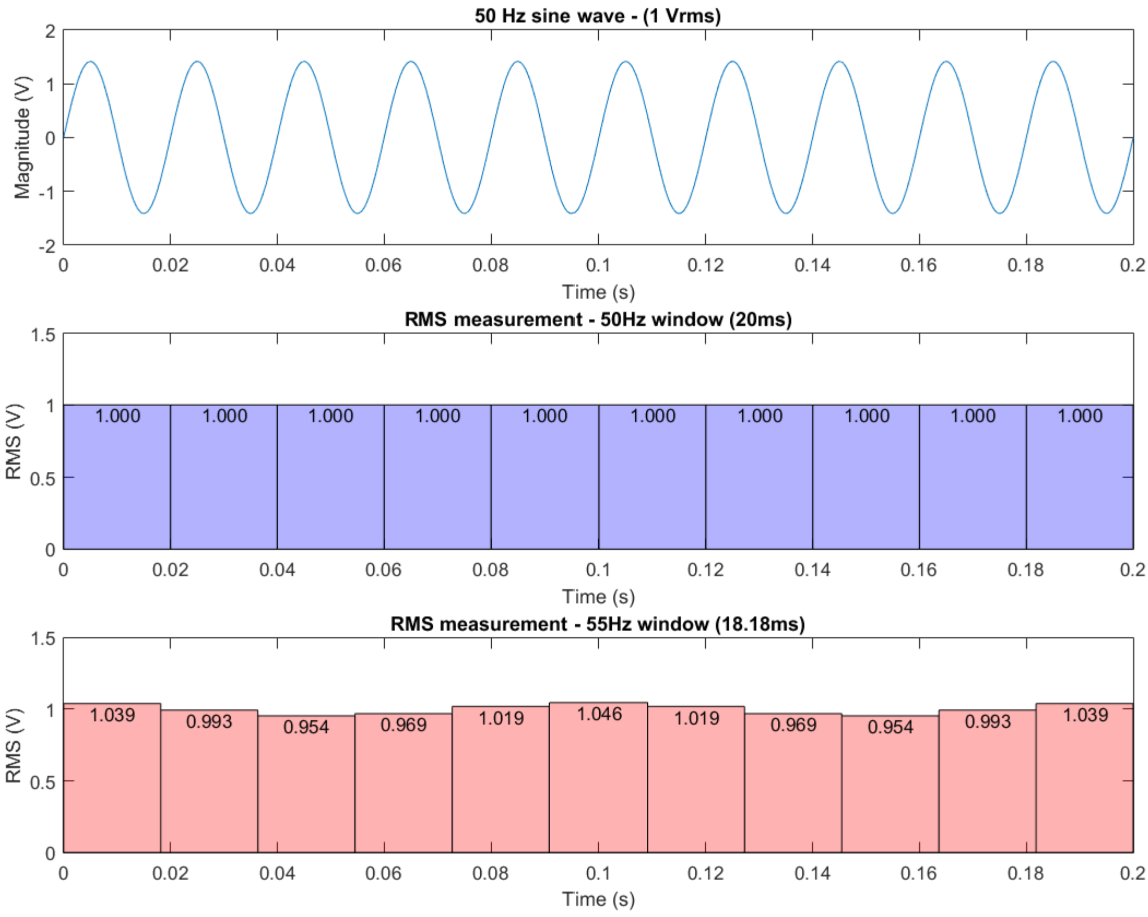

Figure 5:An illustration of RMS error resulting from an incorrectly sized measurement window.

Non-synchronous sample windows clearly introduce a modulation of measured values that will only become correct after a considerable period of time-averaging.

3 EFFICIENCY MEASUREMENTS

Having considered measurement issues associated with any periodic waveform, we now look at the headline subject of this document; the efficiency measurement of a system involving two asynchronous periodic waveforms.

Before more detailed analysis, let us first consider the definition of efficiency commonly assumed to be satisfactory for most systems:

$$Efficiency(\%) = \frac{Power_{Out}}{Power_{In}} \times 100$$

The equation uses time-averaged power values and it is generally assumed to be either a DC signal or a periodic waveform with the same frequency, which makes averaging simpler.

3.1 THE ISSUE WITH 'INSTANTANEOUS' POWER

Occasionally, the equation given above is misunderstood and there is the assumption that taking the ratio of instantaneous power measurements at the input and output device can be used to quickly obtain a meaningful efficiency. However, this is not usually true.

Consider the simple case of a synchronised power system with sinusoidal power waveforms at both input and output - the same frequency, but with a phase shift:

$$Power_{In}(t) = \sin^2(\omega t)$$

$$Power_{Out}(t) = \sin^2(\omega t + \theta)$$

Substituting these values into the efficiency equation would yield:

$$Efficiency(t) = \frac{Power_{Out}(t)}{Power_{In}(t)} = \frac{\sin^2(\omega t + \theta)}{\sin^2(\omega t)}$$

It is clear from observation that there will be values of t for which the equation will include a division of zero, which is undefined.

The explanation for this is that there is a finite energy store within the system, which in the case of most electrical systems, is associated with capacitors and/or inductors, that act as a temporary storage medium. At any instant in time, the input maybe charging these energy stores or output power is sourced from them. It is for this reason that we must again utilise a time-average power measurement.

3.2 EFFICIENCY OF PERIODIC WAVEFORMS

The power of periodic systems is usually described using time-averaged power.

The practical difficulty in measuring these averaged values is that again, a measurement window containing discrete samples will need to be used.

$$Efficiency = \frac{\frac{1}{T_b}\int_{T_0}^{T_0+T_b} P_{Out}(\omega_b t)\, dt}{\frac{1}{T_a}\int_{T_0}^{T_0+T_a} P_{In}(\omega_a t)\, dt}$$

The same issues associated with windowing discussions in Section 2 exist, but with the additional complexity that in most cases, the input and output frequencies are asynchronous. That is $\omega_a \neq \omega_b$.

This means that unless there is a highly unlikely situation where one frequency is a harmonic of the other, any attempt to use a common sample window locked to a single period of either the input or output waveform, will guarantee the introduction of potentially significant error.

3.2.1 Beating power measurements

As stated in section 3.2, one window will generally not match the other, however there will be a time period over which both signals could be accurately averaged. This is a time period that contains an integer multiple of half cycles of both waveforms. This is the beat frequency.

Given a simple power waveform, derived from a sinusoidal input through a resistance of 1Ω :

$$P(t) = \frac{V^2(t)}{R} = \sin^2(\omega t) = \frac{1}{2}(1 - \cos(2\omega t))$$

We can plot an example of a beating waveform with a 50Hz and 60Hz signal.

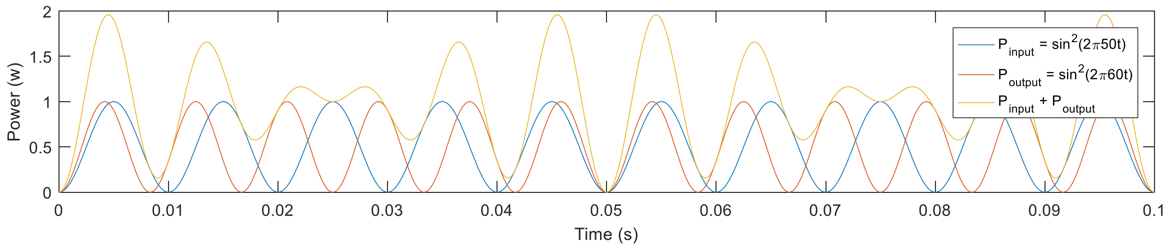

Figure 6: Input and output power, with a summation of the two to aid visualisation of the beat frequency.

The beat frequency[2] is given by $f_{beat} = |f_a - f_b| = |100 - 120| = 20Hz$.

3.2.2 Synchronising to the beat

In theory, the sample window could be synchronised to the beat frequency of the two signals. Using this window would ensure that the two averaged powers and therefore the efficiency measurement is correct. However, there is a significant downside.

The beat frequency is the difference of the two frequencies. Two signals with a similar frequency will result in a very low beat frequency and therefore a large time period for the window. This goes against the aim of quickly obtaining an accurate measurement of the system efficiency.

Figure 7 illustrates how, even as a continuons function, the efficiency oscillates and the measurement only coincides with the true value when the time period is a multiple of the beat frequency.

[2] The frequencies are twice that of the voltage input due to the square term in the power equation.

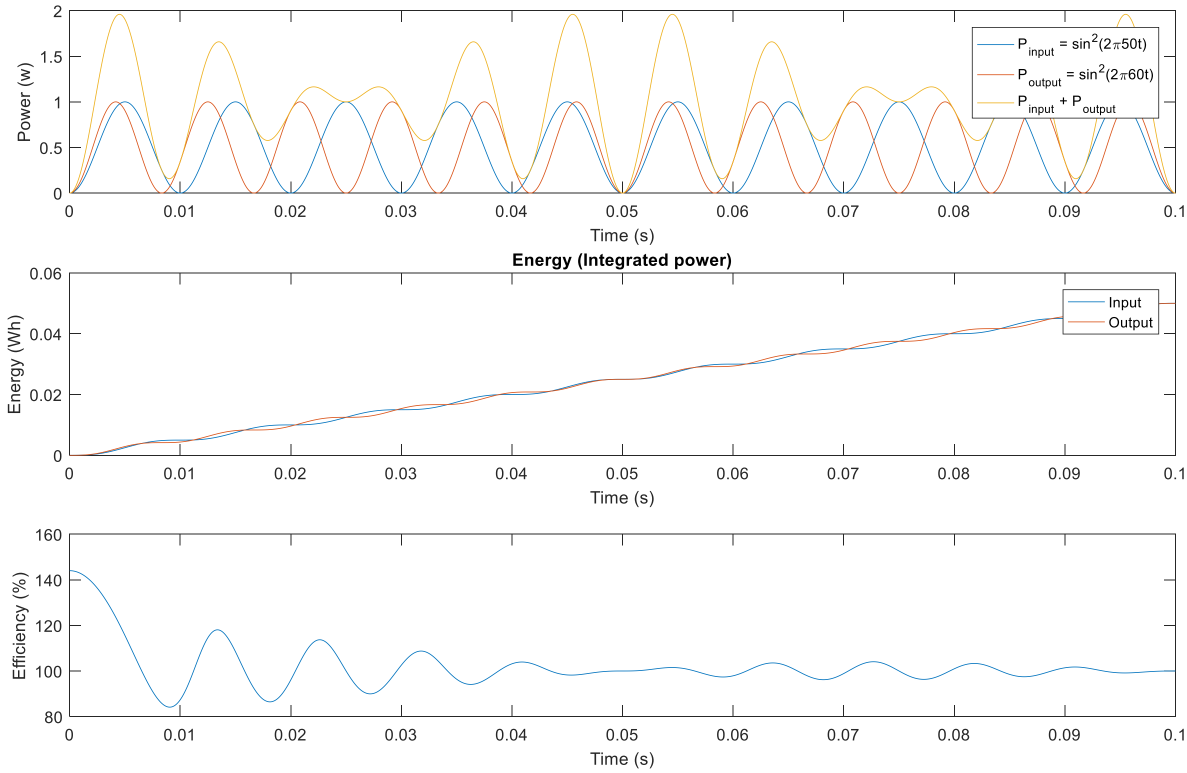

Figure 7: Continuous efficiency calculation using integrated power (energy)

3.3 INDIVIDUALLY SYNCHRONISED TIME WINDOWS

Section 3.2 demonstrated many of the inherent difficulties of obtaining the efficiency in an asynchronous power system.

One method for working with this type of application, which overcomes many of the established issues, is to individually size the measurement window to each power waveform.

The following sub-section provides an example of a system to demonstrate and compare results of the fixed-time windowing to the individually synchronised windowing method.

3.3.1 Frequency converter example

For illustration, imagine a simple and idealised[3] frequency converter with a 50Hz, 1-watt input and 55Hz 1-watt output; this system would have a theoretical efficiency of 100%.

For simplicity, let us assume the equivalent resistance of the input and output are 1Ω. The voltage waveforms can therefore be defined as:

$$v_{In}(t) = \sqrt{2}\sin(2\pi 50\,\mathrm{t})$$

$$v_{Out}(t) = \sqrt{2}\sin(2\pi 55\,\mathrm{t})$$

And the power waveforms:

$$p_{In} = \frac{{v_{In}}^2(t)}{R} = 2\sin^2(2\pi 50\,\mathrm{t})$$

[3] Although not representative of real system, the round values and device with perfect efficiency have been chosen to clearly illustrate the theory.

$$p_{Out} = \frac{v_{Out}{}^2(t)}{R} = 2\sin^2(2\pi 55\ \mathrm{t})$$

For reference, these power waveforms are plotted in Figure 8 plot A.

3.3.2 Fixed-time windowing

Fixed-time windowing uses a common window interval for both the input and output waveform. While the fixed window period may typically be selected for the approximate operating frequency range, in general the period is not directly related to any of the test waveforms.

However, for this example the window size has been fixed to the 50Hz signal and therefore only the 55Hz will have the incorrect working size.

Locking to the 50Hz signal gives a 20ms window. Figure 8 plot B captures the measured average powers for both waveforms. It is clear to see that the incorrectly size windows for the 55Hz output causes the averaged power to oscillate between successive windows.

Figure 8 plot D illustrates how unstable fixed time measurements propagate to an unstable system efficiency computation.

Of course, as discussed in Section 2, given that the power measurements become correct over a longer window size, these measurements can be averaged over multiple windows and will result in a more accurate measurement.

3.3.3 Individually synchronised windows

Using this method, the sample window is sized according to the frequency of each waveform independently. In this example the 50Hz waveform again is sampled using a 20ms window but the 55Hz signal is now sampled using a window of ~18.18ms.

The update rate of the measurement is set by the sample window; as a consequence of choosing different sample windows the power measurements are asynchronous, as can be seen in Figure 8 plot C.

Receiving results at different times is not intuitive but reflects the underlying behaviour of the system being measured. This technique has the additional benefit that the measurements are delivered quickly for both waveforms and provide a more stable measurement.

Given the stability of both input and output power measurements, it is possible to quickly establish system efficiency without the need for filtering and smoothing.

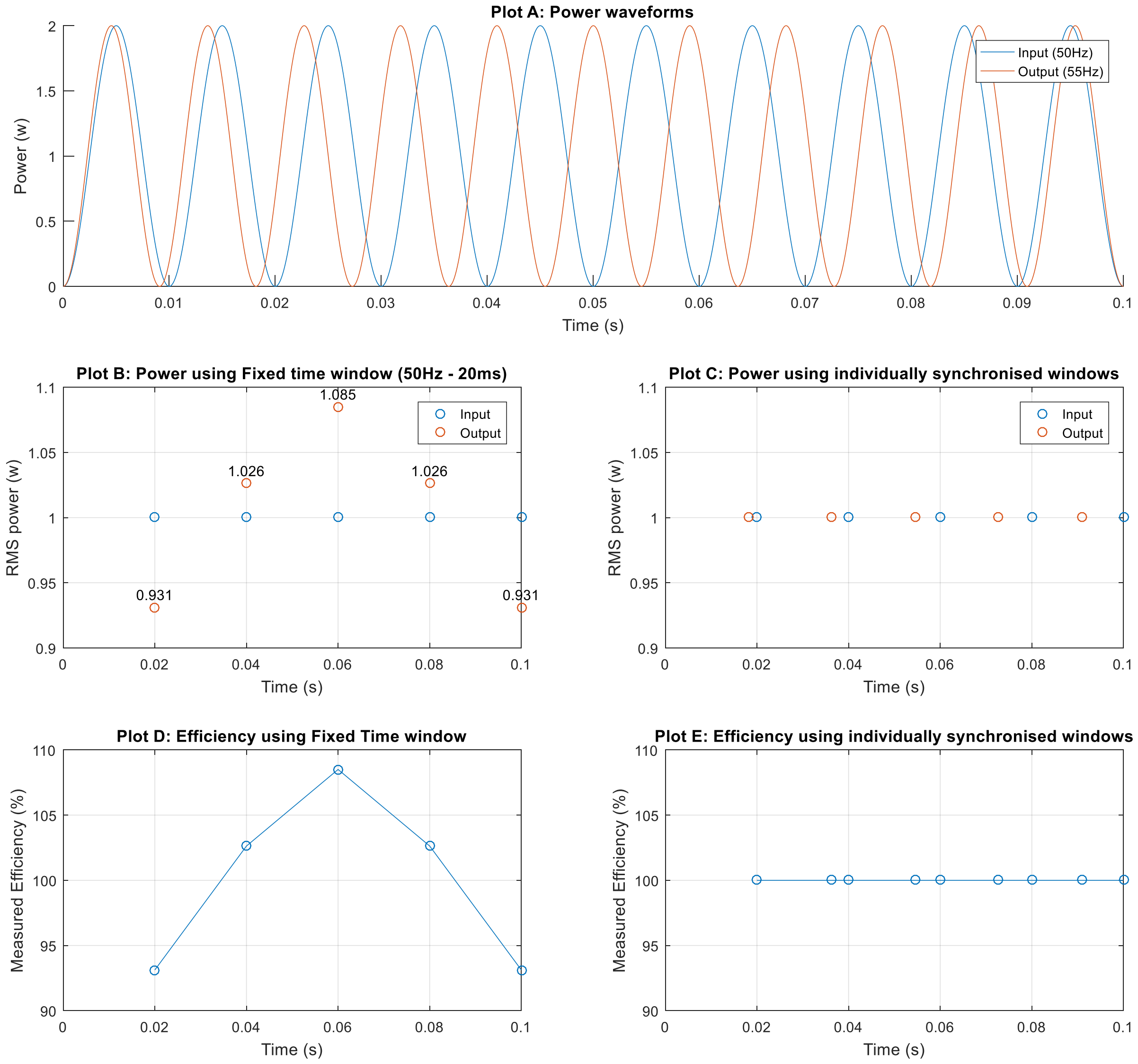

Figure 8: Comparison of fixed-time vs independently synchronised windows.

4 CONCLUSION

Faced with an objective of deriving the efficiency of an electrical system, engineers may instinctively assume that the optimum solution is an analysis of measurement results taken at exactly the same moment in time.

However, by establishing the impact of fixed time measurement windows that are not synchronised with a periodic AC waveform, it becomes clear that simultaneous data acquisition is unlikely to be the optimum solution, unless the application is either steady state or permits an extended period of time averaging.

Ensuring correct frequency synchronisation and a corresponding measurement window with any periodic measurement within a power system, will yield greater measurement stability from which quicker and more accurate efficiency can be derived.